COURSE CODE – 2030501

PG DEGREE EXAMINATION – JAN 2009

M.SC (MATHS)

ALGEBRA – I

(For the Candidates Admitted from Calendar Year 2007)

Time:3 hours **Max.Marks:75**

Section – A

Answer all the Questions: **15 × 1 = 15**

1. Explain the Abelian group.
2. If G is a finite group and H is a subgroup G then O(H) is a ……….. Of O (G).
3. Explain one to one mapping.
4. State cay ley's theorem.
5. What is the order of an n-cycle?
6. Give an example of a non-abelian group.
7. If a, b, c, d ε R, evaluate (a + b) (c + d).
8. The homomorphism φ of R into R^1 is a…..if and only if I (φ) = (0).
9. What is (a / b) + (c / d)?
10. Determine the prime elements in J(i).
11. Define the derivative $f^1(x)$ of the polynomial.
12. If F is a field then $F(x_1 \ldots\ldots x_n)$ is a unique…………domain.
13. In a vector space show that α (v - w) = αv -αw.
14. Define orthonormal set.
15. If A and B are sub modules of M prove A∩B is a sub module of M

Section-B

Answer any Five Questions: **5 × 6 = 30**

16. Prove that if G is an abelian group, then for all a, b ε G and all integers n, $(a.b)^n = a^n . b^n$.

(Or)

b) Let φ be a homomorphism of G onto $\overline{G}$ with kernel K.Then $G/K \approx \overline{G}$.

17. a) Every permutation is the product of its cycles.

(Or)

b) If P is a prime number and P^α / O (G), then G has a subgroup of order P^α.

18. a) Finite integral domain is a field.

(Or)

b) If φ is a homomorphism of R into R^1 then (i) φ(0) = 0, (ii) φ(-a) = -φ (a) for every a ε R

19. a) Let R be a Euclidean ring .suppose that for a, b, c ε R, a ⁄ be but (a, b) = 1then a ⁄ c.

(Or)

b) If f(x) and g(x) are primate polynomials, then f (x) g (x) is a primitive polynomial.

20. a) Prove that the intersection of two subspaces of V is a subspace of V.

(Or)

b) If u, v ε V then $|(u, v)| \leq \|u\| \|v\|$

Section – C

Answer any Two Questions: **2 × 15 = 30**

21. Every group is isomorphic to a subgroup of A(S) for some appropriate S.
22. Every integral domain can be imbedded in a field.
23. Unique factorization Theorem.
24. If V and W are of dimensions m and n respectively over F, then Him (V, W) is of dimension mn over F.
25. If φ is a homomorphism of G into $\overline{G}$ with kernel K, then K is a normal subgroup of G.