COURSE CODE – 2030507

PG DEGREE EXAMINATION – JAN 2009

M.SC (MATHS)

REAL ANALYSIS

(For the Candidates Admitted from Calendar 2007 onwards)

Time:3 hours **Max.Marks:75**

Section – A

Answer all the questions: **15 $\times$ 1 = 15**

1. Definition of derivatives.
2. If ‘f’ takes only positive values in (a ,b) then f is strictly……….on (a , b).
3. Assume f^1exists and is monotonic on an ………then f^1 is continuous on (a , b).
4. Write the Riemann stieltjes integral.
5. Given a < c < b, Define a on (a , b) as follows the values…………are arbitrary.
6. Define lower stieltjes integral.
7. We say that f satisfies Riemann’s condition with respect to α on ……….if, for every y $\in$ >0.
8. Assume that α on (a , b) .If f $\in$ R(α) on (a , b) then $f^2 \in R(\alpha)$ on…………..
9. Write the necessary conditions for existence for Riemann stieltjes integrals.
10. Define Telescoping series.
11. Define outer measure.
12. Write the example for monotone convergence theorem.
13. Write the properties of the Lévesque integral
14. State Riesz-Fischer theorem.
15. State Levi theorem for upper functions.

Section-B

Answer any Five Questions: **5 $\times$ 6 = 30**

16. a) If f is differentiable at c, then f is continuous at c.

(Or)

b) State and prove mean value theorem.

17. a) If f is monotonic on (a , b) then the set of discontinuities of f is countable.

(Or)

b) Let f be defined on (a , b) .then f is of bounded Variation on (a , b) if, and only if f can be expressed as the difference of two increasing functions.

18. a) State and prove Euler’s summation formula.

(Or)

b) If f is continuous on (a , b) and if α is of bounded variation on (a , b) ,then f $\in$ R(α) on (a , b).

19. a) If f is a measurable function and f = g a .c , then g is measurable.

(Or)

b) If f = (f_1…..f_n) be a vector valued functi0obn with a continuous derivative f on (a, b) prove that the curve described by f has length

$$\text{at}\,(a\,,\text{b}) = \int_a^b \left\| \text{f}^{\,1}(t) \right\| \text{dt}\,.$$

20. a) Let u, v, u_1 and v_1 be function in U(I) such that u – v = u_1 – v_1 then $\int_I u - \int_I v = \int_I u_1 - \int_I v_1$.

(Or)

b) If f and g are in L(I) then so are the functions f^+ , f^- , $|f|$ max (f, g) and min(f ,g) moreover we have $\left|\int_I f\right| \le \int_I |f|$

Section – C

Any Two Questions: **2 × 15 = 30**

21. State and Generalized mean value theorem.

22. Assume that c ∈ (a ,b) . If two of the three integrals in 1) exist, then the third also exists and we have $\int_a^c f\, d\alpha + \int_c^d \text{f}\ \text{d}\alpha = \int_a^b \text{f}\ \text{d}\alpha$.

23. Let α be of bonded variation on (a , b) and assume that f ∈ R(α) on (a , b) .then f ∈ R(α) on every subinterval (c , d) of (a , b).

24. Prove that the outer measure of an integral is its length.\

25. State and prove Riesz-Fischer theorem.