

COURSE CODE – 2030503

PG DEGREE EXAMINATION – JAN 2009

MSC (MATHS)

DIFFERENTIAL EQUATIONS WITH HIGHER ORDERS

(For the Candidates Admitted from Calendar 2007 onwards)

Time:3 hours

Max.Marks:75

Section – A

Answer all the questions:

15 × 1 = 15

1. The function ϕ which is zero for all x is also a solution the..... of $L(y) = 0$.
2. State Sturm comparison theorem.
3. The polynomial in λ of degree n , $\det(\lambda E - A)$ is called the.....
4. Define linear homogeneous systems.
5. Write down the variation of constants formula.
6. Write down the Green's formula.
7. Let ϕ_1, \dots, ϕ_n be n solutions of $L(y) = 0$ on an interval.....
8. Define Bessel equation. to the solution.
9. Write the first two approximations ϕ_0, ϕ_1 .
10. Write the properties of limit.
11. Define linear independent.
12. Write down the Wronskian formula.
13. The n solutions of $L(y) = ?$.
14. State the existence theorem.

15. Compute one linearly independent solution.

Section-B

Answer any Five Questions:

5 × 6 = 30

16. a) State and prove Uniqueness theorem.

(Or)

b) If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point X_0 then $W(\phi_1, \phi_2)(x) = e^{-\alpha(x-x_0)} W(\phi_1, \phi_2)(x_0)$.

17. a) There exist n linearly independent solutions of $L(y) = 0$ on

(Or)

b) Find two linearly independent solutions of the equation

$$(3x-1)^2 y^{(1)} + (9x-3)y' - 9y = 0.$$

18. a) State and prove Green's formula.

(Or)

b) The characteristic polynomial for A is given by

$$f(\lambda) = \lambda^n + a_1 \lambda^{n-1} + \dots + a_n.$$

19. a) State and prove Picard's theorem.

(Or)

b) In π_n has only the trivial solution, Green's function G^+ for

$$\pi_n^+ \text{ is given by } G^+(t, \tau) = \bar{G}(\tau, t).$$

20. a) If $A_{24} \neq 0$, the difference $\sigma_m - S_m$ tends to zero as $m \rightarrow \infty$, uniformly on $0 \leq t \leq \pi$.

(Or)

b) If necessary and sufficient that $z = \infty$ be a regular insular point of the equation.

Section – C

Answer any Two Questions:

$2 \times 15 = 30$

21. For any real x_0 , and constants α, β there exists a solution ϕ of the initial value Problem on $-\infty < x < \infty$.

22. Let ϕ_1, \dots, ϕ_n be n solutions of $L(y) = 0$ on an interval I , and let X_0 be any point in I , then

$$W(\phi_1, \dots, \phi_n)(x) = \exp \left[- \int_{x_0}^x a_1(t) dt \right] W(\phi_1, \dots, \phi_n)(x_0).$$

23. If ϕ is a fundamental matrix for $L(h)$, then the function ϕ defined by

24. Let γ be nonnegative continuous and of period I ,

If $\int_0^1 \gamma(t) dt \leq 4$ show that $x'' + \gamma(t)x = 0$ Stable solutions on $(-\infty, \infty)$.

25. Show that J_1^1 satisfies the Bessel equation of order one

$$x^2 y'' + xy' + (x^2 - 1)y = 0.$$